A VMD Based, Nonet and SU(3) Symmetry Broken Model For Radiative Decays of Light Mesons

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1. Introduction

It is a long standing problem to define a framework in which all radiative decays of light flavor mesons can be accurately accounted for. A few kinds of different models have been proposed so far. The most popular modelling is in terms of magnetic moments of quarks [1–3], which includes some kind of SU(3) breaking effects by having a magnetic moment for the s quark, slightly different to that of the d quark. SU(2) symmetry breaking is implemented in some way, as the u and d magnetic moments are not exactly in the ratio of the corresponding quark charges [4].

Another traditional approach is to use SU(3) relations among coupling constants [5]. This yields reasonable descriptions of radiative decays [6], though the success is not complete. The O'Donnell model assumes exact SU(3) flavor symmetry, while nonet (or U(3) flavor) symmetry is explicitly broken. As it follows from a quite general conceptual framework, this model is widely independent of detailed dynamical properties and assumptions, except for its assumption of unbroken SU(3) symmetry, of course. *A priori*, this model depends on two mixing angles and, because of its breaking nonet symmetry, it also depends on three coupling constants (instead of one, if nonet symmetry were to hold). This model covers all couplings like $PV\gamma$ but lacks to describe $P\gamma\gamma$ decays which remain unrelated.

Recently, several models have been proposed [7–9], motivated by effective Lagrangian approaches to the interactions of vector mesons [10,11], and including SU(3) symmetry breaking as per Bando, Kugo and Yamawaki (hereafter referred to as BKY) [12,13]. More recently, a new kind of model has been proposed [14], where additional symmetry breaking effects are introduced by means of the (measured) leptonic decay constants of vector mesons. The study of radiative decays of light flavor mesons is also connected with the long standing problem of η/η' mixing [6,9,15,16] and to its possible association with a glue component inside light mesons [14,17]. Recent developments advocate a more complicated η/η' mixing scheme [18,19], which has received support from some phenomenological analyses [21,22]. Another approach in the same vein has been proposed quite recently by Escribano and Frère [23]. As effects of SU(3) symmetry breaking are clearly observed in the data on radiative decays of light mesons [6,14], they have surely to be accounted for. We do it following the BKY breaking mechanism [12,13]. In this way, by means of the FKTUY Lagrangian and of the BKY breaking scheme, we can construct a Lagrangian formulation of the O'Donnell model and extend it to the case where the SU(3) flavor symmetry is broken. This additionally provides an algebraic connection between $VP\gamma$ and $P\gamma\gamma$ coupling constants.

What is presented here is mainly an account of a work [24] devoted to a to the study of radiative and leptonic decays of light flavor mesons within the general VMD framework represented by the hidden local symmetry model [10] (hereafter referred to as HLS) and its anomalous sector [11] (hereafter referred to as FKTUY). All formulae and technical details, which could be skipped here, can be found in Ref. [24]. Some progress has been made however in the qualitative understanding of the problem and of the solutions proposed there ; this will reflect here by a somewhat different perspective.

This work proposes a new model based on the approach of the HLS model which describes the mutual interactions of the electrom agnetic field, the vector and pseudoscalar mesons ; we will concentrate here mainly on its anomalous FKTUY sector. A careful study of the HLS model, also in its anomalous sector (the FKTUY Lagrangian), is motivated by the successful description [25] provided by the HLS model of all published data on the pion form factor [26]. This success has been recently confirmed with the new data set of the CMD-2 Collaboration [27,28]. Moreover, as shown in Ref. [24], leptonic decays of vector mesons gives an additional support to this model in the non-anomalous sector. Namely, the basic HLS parameter a extracted from the pion form factor ($a \simeq 2.4$), appears to be in good agreement with radiative and leptonic decays of light mesons, where it is found $a \simeq 2.5$, instead of $a = 2$ as expected from standard VMD [10,29]. The HLS model is an expression of the Vector Meson Dominance (VMD) assumption [29] ; it thus gives a way to relate the radiative decay modes $VP\gamma$ to each other and to the $P\gamma\gamma$ decays for light mesons, by giving a precise meaning to the equations shown in Fig. 1. One can try to estimate naively this relation by means of the results from recent fits [30] to the cross sections $e^+e^- \to \pi^0\gamma$ and $e^+e^- \to \eta\gamma$, and by using other information collected in the Review of Particle Properties [31]. The results given in Table ?? are quite impressive ; this is indeed not a fit, but mere algebra. Thus, all systematics can pill up and, moreover, the meson masses used in order to estimate the propagators at $s = 0$ are simply the (Breit–Wigner) accepted masses [31] ; this is surely a very crude assumption, at least for the ρ^0 meson. However, t his exercise teaches us that the central hint of the Vector Meson Dominance assumption is sharply grounded and this motivates to try going beyond as much as possible.

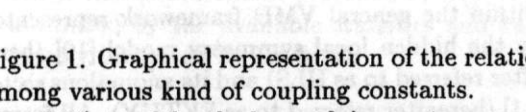

Figure 1. Graphical representation of the relation among various kind of coupling constants.
V and V' stand for the lowest lying vector mesons (ρ^0, ω, ϕ); the internal vector meson lines are propagators at $s = 0$ and are approximated by the corresponding tabulated [31] masses squared.

A further comment is of relevance concerning the VMD prediction for the π^0 decay width. Actually, the two–photon width of the π^0 can be computed, as sketched in Fig. 1, from two different ways since the basic VVP diagram is $\pi^0\rho^0\omega^I$, where ω^I is the ideal (purely non–strange) combination of the (physical) ω and ϕ fields. A first estimate is thus obtained from the coupling $\pi^0\rho^0\gamma$ (here the hidden vector meson line is surely ω^I) and is 12.79 ± 2.59 eV ; a second estimate is obtained from using instead the couplings $\pi^0\omega\gamma$ and $\pi^0\phi\gamma$ (here the hidden vector meson line is surely ρ^0 for both) and is 8.86 ± 0.29 eV. The qualitative difference of both estimates reflects problems with the ρ mass definition which will not be examined here. What is given in Table 1 is simply the mean value of both estimates.

2. An Exact SU(3) Symmetry Framework

The formalism which describes the decays $V \to P\ \gamma$ and $P \to V\gamma$ within an exact SU(3) flavor symmetry framework has been given by P. O'Donnell [5]. The corresponding decay amplitudes can be quite generally written as

$$T = g_{VP\gamma}\epsilon_{\mu\nu\rho\sigma}k^\mu q^\nu \varepsilon^\rho(V)\varepsilon^\sigma(\gamma) \quad (1)$$

using obvious notations. This expression can be found by relying entirely on gauge invariance and does not require the help of any specific Lagrangian.

Using SU(3) symmetry, the coupling strengths $g_{VP\gamma}$ between physical vector and pseudoscalar mesons in radiative decays are expressed in terms of two angles (θ_V and θ_P) which describe the mixtures of singlet and octet components, and of three coupling constants ($g_{V_8P_8\gamma}$, $g_{V_1P_8\gamma}$ and $g_{V_8P_1\gamma}$); indeed, assuming that the photon behaves like an SU(3) octet cancels out the possible coupling $g_{V_1P_1\gamma}$. We do not reproduce here the expressions for the $g_{VP\gamma}$ in terms of the elementary couplings $g_{V_iP_j\gamma}$ and the mixing angles ; they can be found in Ref. [5] and in Appendix A7 of Ref. [6], where a misprint has been corrected. These formulae use mixing angles describing deviations from ideal mixing, introduced long ago in Ref. [32]. The vector meson field matrix V is usually written [1] :

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} (\rho^0 + \omega^I)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (-\rho^0 + \omega^I)/\sqrt{2} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\phi^I \end{pmatrix} . \quad (2)$$

in terms of ideally mixed states (ω^I, ϕ^I). Correspondingly the pseudoscalar field matrix is usu-

[1] The sign in front of ϕ^I means that we define $\phi^I = -|s\bar{s}>$.

ally defined as :

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\pi_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}, \tag{3}$$

using the conventional octet and singlet components (π_8, η_0) for the isoscalar mesons. The physical states (ω, ϕ, η, η') are generated from ideally mixed states by means of standard rotations of angles δ_V or δ_P for resp. vector and pseudoscalar mesons. Correspondingly, the rotation angles from singlet and octet states to the physically observed mesons are traditionally named θ_V and θ_P. These well known relations can be found in Refs. [5,13,24,31]. With these definitions for the field matrices, the effective FKTUY Lagrangian which describes the anomalous sector of the HLS model is [11]

$$L = -\frac{3g^2}{4\pi^2 f_\pi} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}[\partial_\mu V_\nu \partial_\rho V_\sigma P]. \tag{4}$$

The parameter g is the universal vector meson coupling and is tightly related to the coupling of the ρ meson to a pion pair. $f_\pi = 92.41$ MeV is the pion decay constant. The partial widths for all $VP\gamma$ and $P\gamma\gamma$ modes are derived herefrom, using also the $V\gamma$ transition amplitudes and the expressions for the vector meson masses given in the standard (non–anomalous) HLS Lagrangian [10,11] by

$$\begin{aligned} L = & \cdots + \tfrac{1}{2} a f_\pi^2 g^2 [(\rho^0)^2 + (\omega^I)^2 + (\phi^I)^2] - \\ & -aef_\pi^2 g\, [\rho^0 + \tfrac{1}{3}\omega^I + \tfrac{\sqrt{2}}{3}\phi^I].A + \cdot \end{aligned} \tag{5}$$

expressed in terms of ideally mixed states and of the electromagnetic field A. One should note the occurence of the HLS parameter a expected from standard VMD to fulfill $a = 2$, while recent fits of the pio n form factor [25,27,28] indicate a preferred value $a \simeq 2.4$. The coefficient in Rel. (4) is fixed [11] in order that this Lagrangian, supplemented with the relevant information in Rel. (5), leads to the usual expression for the amplitude of $\pi^0 \to \gamma\gamma$.

At this point, it should be emphasized that exact SU(3) symmetry is not. in conflict with releasing the condition of nonet symmetry (which corresponds to the stronger U(3) symmetry) usually stated in effective Lagrangian models for both the vector and pseudoscalar meson sectors [9–11,33,34]. The main problem concerns the part of the Lagrangian which gives the kinetic energy and mass terms of the pseudoscalar singlet field. We will comment more on this point below, but will not try any approach to this specific problem. The field matrices above can be written $V = V_8 + V_1$ and $P = P_8 + P_1$, which exhibits their (matrix) octet and singlet mixtures. It can be checked, that the O'Donnell model [5,24] can be generated by simply replacing in Rel. (4) the (nonet symmetric) vector and pseudoscalar field matrices by

$$\begin{cases} P = P_8 + xP_1 \\ V = V_8 + yV_1 \end{cases} \tag{6}$$

In this case, we have

$$\begin{cases} g_{V_8P_8\gamma} = & G = -\dfrac{3eg}{8\pi^2 f_\pi} \\ g_{V_1P_8\gamma} = & yG \\ g_{V_8P_1\gamma} = & xG \end{cases} \tag{7}$$

and the mixing angles, as free parameters to be determined from fit to data. This identification is already interesting, as it relates the main coupling constant G in the O'Donnell model to more usual quantities (g, f_π). Finally, x and y are the deviations from nonet symmetry in respectively the pseudoscalar and vector sectors.

It was phenomenologically checked[2] [6], that radiative decays of light mesons are consistent with nonet symmetry in the vector sector; thus, data accommodate $y = 1$ quite naturally. However, the same analysis concluded to a small but significant departure (slightly more than 4σ) from nonet symmetry in the pseudoscalar sector, which corresponds to $x \simeq 0.90$. From a physics point of view, the resulting picture was close to be acceptable, as only two decay modes, $K^{*0} \to K^0\gamma$ and $\phi \to \eta\gamma$, were not satisfactorily accounted for (see in Ref. [6] the "internal fit" entry of Table 8). Qualitatively, the former disagreement could be due to SU(2) symmetry breaking because of the K^{*0} quark content[3]. However, the disagreement about the later mode (more than a factor of 2) is clearly a signal of unaccounted for SU(3) breaking effects, since the branching fraction for $\phi \to \eta\gamma$ has been recently confirmed twice [35,36] at the VEPP-2M collider. Moreover, the "internal fit" entry in Table 8 of Ref. [6] shows a good prediction for $\phi \to \eta'\gamma$, if one refers to the recent measurements of this mode, still at VEPP-2M, by the CMD2 [37] and SND [38] experiments.

Therefore the O'Donnell model [5,6] is already close enough to observations that one may con-

[2] The quoted deviations of x and y from unity have been confirmed by t he present analysis. For instance, releasing y in the present U(3), SU(3) broken model leads to $y = 0.996 \pm 0.033$, quite consistent indeed with $y = 1$ and thus with nonet symmetry.

[3] However, a factor of 1.5 at the rate level, *i.e.* a factor 1.25 at the level of coupling constants, could look somehow beyond expectable SU(2) breaking effects.

clude that nonet symmetry breaking is a working concept and that only some amount of SU(3) breaking is needed in order to achieve a quite consistent description of radiative decays. This is the main purpose of the work presented here, and we are to be valuably helped by the connection which can be done between O'Donnell model and the HLS approach [10,11], as illustrated above. It should be re–emphasized, however, that the original model of O'Donnell did not relate $VP\gamma$ and $P\gamma\gamma$ modes, while the connection just sketched of this model with the HLS approach provides the lacking algebraic connection.

In the approach just sketched, we actually assume the existence of only one singlet state for pseudoscalar mesons, constituted of u, d and s quarks and their antiquarks. The modifications of the P matrix performed in order to reconstruct a SU(3) invariant, with broken U(3) model from the FKTUY Lagrangian [11], implies that the singlet component can be accounted for in the non–anomalous Lagrangian. This difficult question is intimately related to the $U(1)_A$ anomaly, (see Refs. [17,39,18,19] for recent accounts) and we cannot pretend to solve it within the present context. We shall only assume that there exists a way to accommodate satisfactorily the singlet part in the non–anomalous Lagrangian, namely its mass term and its kinetic energy term (which is affected by the parameter x introduced above). Moreover, the problem should be qualitatively different if breaking of nonet symmetry could be fully replaced by an additional singlet state.

Recent theoretical developments indeed tend to advocate that the singlet sector of pseudoscalar mesons could well be not saturated [14,17] by the standard singlet $\eta_0 = (u\overline{u}+d\overline{d}+s\overline{s})/\sqrt{3}$ only. One (or more) of the glueballs predicted by QCD could play a non–negligible role. With this respect, a part of the broad structure named [31] $\eta(1440)$ is still controversially considered as a glueball candidate (see the minireview in Ref. [31]). On the other hand, following the measurement at CLEO [40] of an unexpectedly high rate of $B \to \eta' K$, a $c\overline{c}$ component in the η' meson is sometimes considered (see Refs. [41,21] and references quoted therein). We shall discuss this question at the relevant place, and its relation with nonet symmetry breaking.

3. SU(3) Breaking of the HLS–FKTUY Model

SU(3) symmetry breaking of the HLS Lagrangian originates from Refs. [10,12]. A comprehensive account of the HLS model is given in Ref. [10]. More globally, a recent review of VMD can be found in Ref.[29]. Finally, briefs accounts and some new developments can be found in Refs. [7,8,13,33], connected more precisely with the anomalous sector [11].

3.1. SU(3) Breaking Mechanism of the HLS Model

Basically, the SU(3) breaking scheme we use has been introduced by Bando, Kugo and Yamawaki [12] (referred to as BKY) and has given rise to a few variants [7,13], as well as to extension to SU(2) breaking [8]. We refer the reader to Refs. [7,12,13] for detailed analyses of the properties of known variants of the BKY breaking scheme. In the following, we use basic consequences common to the original BKY mechanism [12], its hermitized variant and the so-called new scheme, both discussed in Ref. [13]. In these cases, SU(3) symmetry breaking defines a renormalized pseudoscalar field matrix P' in terms of the bare one P given above by

$$P' = X_A^{1/2} P X_A^{1/2}, \tag{8}$$

where the breaking matrix X_A writes $\mathrm{diag}(1,\ 1,\ 1+c_A)$ and we have [12,13]

$$\ell_A \equiv 1 + c_A = \left(\frac{f_K}{f_\pi}\right)^2 = 1.495 \pm 0.030\ . \tag{9}$$

It should be noted [13], that the field renormalization (Eqs. (8) and (9)) is requested in order to recover the charge normalization condition, $F_{K^+}(0) = 1$, expected for the kaon form factor $F_{K^+}(s)$, even in presence of SU(3) breaking. The numerical value just g iven is deduced from the experimental information quoted in Ref. [31]. Concerning vector mesons, beside changing the couplings of K^* and ϕ mesons to pseudoscalar pairs, SU(3) breaking modifies the vector meson mass terms and their coupling to the electromagnetic field in the following way

$$\begin{array}{ll} L = & \frac{1}{2} a f_\pi^2 g^2 \left[(\rho^0)^2 + (\omega^I)^2 + \ell_V (\phi^I)^2\right] - \\ & -a e f_\pi^2 g \left[\rho^0 + \frac{1}{3}\omega^I + \ell_V \frac{\sqrt{2}}{3}\phi^I\right] A \end{array} \tag{10}$$

We have defined $\ell_V \equiv (1 + c_V)^2$. We refer to the BKY breaking in the vector sector as X_V breaking. These notations (X_A and X_V) are motivated by the fact that the BKY breaking scheme [12,13,7] results in breaking matrices (X_A and X_V) which come included in trace operators. We have $X_V = \mathrm{diag}(1,\ 1,\ 1+c_V)$. Properties of this parameter c_V can be found in Refs. [12,13,24]. We can reexpress the HLS Lagrangian in terms of physical field combinations, by rotating the ideal fields by an angle δ_V. The Lagrangian piece given in Eq. (10) thus becomes

$$\begin{array}{rl} L = & a f_\pi^2 g^2 \left[(\rho^0)^2 + (\cos^2\delta_V + \ell_V \sin^2\delta_V)\omega^2 + \right. \\ & \left. +(\sin^2\delta_V + \ell_V \cos^2\delta_V)\phi^2 \right] + \\ & + a f_\pi^2 g^2 \ell_V \omega.\phi - \\ & -ae f_\pi^2 g \left[\rho^0 + \frac{1}{3}(\cos\delta_V + \ell_V \sqrt{2}\sin\delta_V)\omega - \right. \\ & \left. -\frac{1}{3}(\sin\delta_V - \ell_V\sqrt{2}\cos\delta_V)\phi \right] .A \end{array} \quad (11)$$

The coefficients affecting the ρ, ω and ϕ fields in the last term, are commonly denoted $-ef_{\rho\gamma}$, $-ef_{\omega\gamma}$ and $-ef_{\phi\gamma}$. They are estimated from the vector meson decay widths to e^+e^- by

$$\Gamma(V \to e^+e^-) = \frac{4\pi\alpha_{em}^2}{3m_V^3}|f_{V\gamma}|^2 \quad (12)$$

One still observes, as in the unbroken case, the occurence of the HLS parameter a. One should also note the occurrence of a direct transition term $\omega.\phi$, generated by the rotation δ_V. This term plays an important role when computing some matrix elements [24].

3.2. A Phenomenological Lagrangian for Radiative Decays

Following FKTUY [11], the anomalous U(3) symmetric Lagrangian describing PVV interactions and, together with Eqs. (5), $PV\gamma$ and $P\gamma\gamma$ transitions is given by Eq. (4). It full expansion can be found in Ref. [13]. Postulating that the same formulae apply when breaking nonet symmetry ($x \neq 1$), is confirmed by its formal agreement with the O'Donnell derivation of the coupling constant formulae. When breaking SU(3) symmetry à la BKY [12], the vector meson part of the non–anomalous HLS Lagrangian, of relevance for our purpose, is given by Rels.(10) or (11). However, breaking the SU(3) symmetry à la BKY, also implies that we have to reexpress the FKTUY Lagrangian in terms of the renormalized matrix P', instead of the bare one P; this is done using Rel. (8), with the (fixed) parameter given in Rel. (9). We remind that P' is also modified by the replacement $\eta_0 \longrightarrow x\eta_0$. Propagating this field renormalization down to the FKTUY Lagrangian writes

$$L = -\frac{3g^2}{4\pi^2 f_\pi}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\partial_\mu V_\nu \partial_\rho V_\sigma X_A^{-1/2} P' X_A^{-1/2}]. \quad (13)$$

Then, the VVP Lagrangian is changed in a definite way by the symmetry breaking parameter ℓ_A defined above (see Eq. (9)) and supposed to have a well understood numerical value (practically 1.5). The expanded f orm of this Lagrangian is given in the Appendix of Ref. [24]. In principle, from this Lagrangian and the non–anomalous L_V Lagrangian piece given in Eq. (11), one is able to construct the decay amplitudes for the $V \to P\gamma$, $P \to V\gamma$, $V \to e^+e^-$ and $P \to \gamma\gamma$ processes. The coupling constants for the decays $V \to P\gamma$ and $P \to V\gamma$ are given by Eqs. (A.1) and (A.2) in the Appendix. They are related to the partial decay widths through

$$\left\{ \begin{array}{l} \Gamma(V \to P\gamma) = \dfrac{1}{96\pi}\left[\dfrac{m_V^2 - m_P^2}{m_V}\right]^3 |G_{VP\gamma}|^2 \\ \Gamma(P \to V\gamma) = \dfrac{1}{32\pi}\left[\dfrac{m_P^2 - m_V^2}{m_P}\right]^3 |G_{VP\gamma}|^2 \end{array} \right. \quad (14)$$

The main coupling G of the O'Donnell model is given by the first Rel.(7). It should be noted that the expression for the coupling constants, when SU(3) breaking is turned off, coincide with the original O'Donnell formulae [5] with $y = 1$; they are obtained continuously by making $Z \to 1$, or equivalently $f_K \to f_\pi$. Some of these couplings are totally unaffected by any breaking process (such as $G_{V\pi\gamma}$), while some are affected only by broken nonet symmetry (like $G_{\rho^0\eta\gamma}$ and $G_{\rho^0\eta'\gamma}$). Couplings involving both isoscalar vector and pseudoscalar mesons are instead affected in a non–trivial way. For instance, the coupling constants for decays involving η and η' mesons are not simply rescaled in the breaking procedure, but treated differently with respect to their strange and non–strange contributions. In order to compare with recent modellings, we see for instance that the relation $G_{\rho^0\eta\gamma}/G_{\rho^0\eta'\gamma} = \tan\delta_P$ is sharply modified by nonet symmetry breaking. It is interesting to note that the FKTUY Lagrangian [11], broken as we propose, expresses all radiative coupling constants in terms of f_π, f_K, x and of the two mixing angles θ_P, θ_V.

3.3. The VMD Description of $\eta/\eta' \to \gamma\gamma$ Decays

Using standard rules, the same Lagrangian information allows to reconstruct definite expressions (see Eqs.(A.4)) for the two–photon couplings of the pseudoscalar mesons; computations are straightforward, even if somehow heavy. These couplings relate to partial widths by

$$\Gamma(X \to \gamma\gamma) = \frac{M_X^3}{64\pi}|G_{X\gamma\gamma}|^2 \quad , X = \pi^0,\ \eta,\ \eta'. \quad (15)$$

The expression for $G_{\eta\gamma\gamma}$ in Eqs.(A.4) compares well with the corresponding expression of Ref. [42] deduced from the Nambu–Jona–Lasinio model. This shows that breaking parameters in this reference, originally expressed as functions of effective quark masses, also get an expression in terms of f_π/f_K. More precisely, as remarked in Ref. [42], in the (chiral) limit of vanishing meson masses, their breaking parameter, which can be formally identified to our $Z = [f_\pi/f_K]^2$, is simply the ratio m_q/m_s (q stands for either of u or d which have equal masses if SU(2) flavor symmetry is fulfilled) of the relevant effective masses of quarks. With this respect, a surprising connection could be made with the traditional description of radiative decays using quark magnetic moments [1–3]. Indeed, the present fit values for these are [4] :

$$\mu_u = 1.852 \quad \mu_d = -0.972 \quad \mu_s = -0.630 \qquad (16)$$

in units of Bohr magnetons. These magnetic moments corresponds to the following quark (effective) masses

$$\begin{aligned} m_u &= 355.1 \text{ MeV} \\ m_d &= 337.4 \text{ MeV} \\ m_s &= 522.8 \text{ MeV} \end{aligned} \qquad (17)$$

It is indeed a point that $m_s/m_u = 1.47$, $m_s/m_d = 1.55$ compare well with $[f_K/f_\pi]^2 = 1.495$, as it can be guessed from the remark by Takizawa *et al.* [42]. Whether, this is accidental, or reveals a deeper property is an open question.

3.4. The WZW Description of $\eta/\eta' \to \gamma\gamma$ Decays

More interesting is that, starting from broken HLS and FKTUY, we recover the traditional form for these amplitudes, (*i.e.* the one mixing angle expressions of Current Algebra [15,16,43]). Using these standard expressions, one indeed gets through identification with our Rels. (A . 4)

$$\frac{f_\pi}{f_8} = \frac{5-2Z}{3} \; , \; \frac{f_\pi}{f_1} = \frac{5+Z}{6}x \quad , \qquad (18)$$

where $Z = [f_\pi/f_K]^2$. This shows that, in the limit of SU(3) symmetry, we have $f_8 = f_\pi$ and $f_1 = f_\pi/x$, and that $f_1 = f_8 = f_\pi$ supposes that there is no symmetry breaking at all. Actually, these formulae mean that, instead of going through the whole machinery of VMD by starting from the broken FKTUY Lagrangian, one could get these coupling constants for two–photon decays of the pseudoscalar mesons by starting from the WZW Lagrangian [44,45]. Indeed, this can be written

$$L_{WZW} = -\frac{e^2}{4\pi^2 f_\pi}\epsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu \partial_\rho A_\sigma \text{Tr}[Q^2 P] \qquad (19)$$

where Q = diag(2/3,–1/3,–1/3) is the quark charge matrix and P is the bare pseudoscalar field matrix. Changing to P' through Rel.(8), allows indeed to recover directly (and trivially) the couplings given in the Appendix[4]. Then, the exercise of having derived these coupling constants from the broken FKTUY Lagrangian simply proves that Rels. (A . 1) and (A . 2) are algebraically correct ... This illustrates clearly that, what is named f_8 in the Current Algebra [43] expressions for η/η' decays to two photons, can be expressed solely in terms of f_π and f_K, in a way which fixes its value to $f_8 = 0.82 f_\pi$. The fact that the WZW Lagrangian leads to the same results as the FKTUY Lagrangian simply states their expected equivalence when deriving two–photon decays amplitudes for pseudoscalar mesons.

On the other hand, the SU(3) sector of Chiral Perturbation Theory (ChPT) predicts [15,46] $f_8/f_\pi \simeq 1.25$. One could thus think to a contradiction [24] between VMD (or FKTUY) and WZW on the one hand, and ChPT on the other hand. However, it seems more likely to think to different definitions of pseudoscalar parameters and then to a misleading apparence. Indeed, in the VMD procedure, the expressions for f_8 and f_1 rely on matrix elements for $< \gamma\gamma|L_{WZW}|\eta >$ and $< \gamma\gamma|L_{WZW}|\eta' >$. In ChPT, the corresponding quantities are rather defined through other matrix elements $< 0|\partial^\mu A_\mu^{8,1}|\eta >$ and $< 0|\partial^\mu A_\mu^{8,1}|\eta' >$, where the A's are axial fields of the appropriate flavor.

There are algebraic relations between both sets of definitions. Specifically, the quantities we have denoted f_8, f_1 and θ_P allow to define the two decay constants F_0 and F_8 of Refs. [18,19] and their two mixing angles θ_0 and θ_8. However, within the context of radiative decays $VP\gamma$ and $P\gamma\gamma$, this complicates the picture, without providing a deeper understanding of radiative decays, as will be seen shortly. We shall nevertheless examine this question elsewhere.

4. Fitting Decays Modes with the Broken Model

In this section, we focus on the model for coupling constants given by Eqs.(A.1) to (A.4) and we use them for a fit to radiative decays of light mesons. the corresponding data are all taken[5] from the Review of Particle Properties [31]. In this section, we discuss only the fit information at the level of the model parameters properties and values. The detailed discussion about the predictions for decay rates is postponed to Section 6.

4.1. Comments on Radiative K^* Decay Measurements

From what is reported several times in the literature [4,6,14], one might expect potential problems with one or both K^* decay modes. Therefore, we have followed the strategy of perform-

[4]Actually, these formulae have been already reached in Ref. [13], but they were expressed in terms of c_A, which leads to non–tra nsparent formulae.

[5]We have however used in the fits [24], as partial width for $\eta \to \gamma\gamma$, the mean value of the measurements reported by $\gamma\gamma$ experiments, instead of the PDG [31] value, which is affected by the single existing Primakoff measurement. We shall comment more on this below.

ing fits of all radiative decay modes except for these two. Then, the fit values of the free parameters allow us to predict a value for the partial widths $K^{*0} \to K^0\gamma$ and $K^{*+} \to K^+\gamma$, making it possible to compare the χ^2 distance of each these predicted values to the corresponding measured values [31]. In all fits performed with the model described above, we have found that the prediction for $K^{*0} \to K^0\gamma$ is in fairly good agreement with the corresponding measurement, while the expected value for $K^{*+} \to K^+\gamma$ is always at about 5σ from the accepted value [31]. Therefore, in the fits referred to hereafter, the process $K^{*+} \to K^+\gamma$ has been removed. The difficulty met with this decay mode in several studies mentioned above could cast some doubt on the reliability of this measurement, performed using the Primakoff effect. However, it cannot be excluded that this measurement is indeed correct and that the reported disagreement simply points toward the need of refining the models. Actually, there indeed exists a way to accommodate this measurement [24], at the expense of complicating somehow the model above. It happens that it does not change anything to all the rest ; it is the reason why we prefer working with the simply model described above, where all parameters have an intuitive meaning. We shall comment specifically below on how the process $K^{*+} \to K^+\gamma$ can be accommodated, referring for details to Ref. [24].

4.2. The SU(3) Breaking Parameter ℓ_A and the Value of f_K

The key parameter associated specifically with the breaking of SU(3) flavor symmetry is the BKY parameter ℓ_A, expected to be equal to $[f_K/f_\pi]^2$ (see Rel. (9)) . As starting point in our fit, we have left free all parameters : G, x, θ_V, θ_P and ℓ_A. We thus got a nice fit probability (χ^2/dof= 10.74/9) and the result we like to mention from this fit is

$$\ell_A = 1.480^{+0.049}_{-0.047} \qquad (20)$$

which is almost exactly the value expected from the known ratio f_K/f_π (see Eq. (9)). This gives, of course, a strong support to the breaking mechanism proposed by Bando, Kugo and Yamawaki [12,13]. Indeed, the relation between ℓ_A and f_K/f_π, which is mandatory within the BKY breaking scheme in order to fulfill $F_{K^+}(0) = 1$ even when SU(3) flavor breaking is turned on, is found to hold numerically to quite a nice precision. One could consider the result in Eq. (20) as providing an interesting estimate of f_K/f_π, independent of measurements of K and π decays

$$\frac{f_K}{f_\pi} = 1.217^{+0.021}_{-0.019}. \qquad (21)$$

This result strongly suggests that one can reasonably *fix* $\ell_A = 1.50$ (at its physical value). Then, the single free breaking parameter which influences the coupling constants in radiative decays, beside mixing angles, is the nonet symmetry breaking parameter x.

4.3. The Fit Parameter Values

Therefore, the preliminary fit sketched above allows us to conclude that the only actual free symmetry breaking parameter is x, once we do not consider a coupling of the η/η' doublet to glue. Stated otherwise, except for the two mixing angles, we only have two free parameters to fit the data set, as in the unbroken case [6]. One, named G, is connected with the vector meson universal coupling g, the other is the nonet symmetry breaking parameter x. The former is clearly fundamental (G) while it is uncertain whether or not the latter should be considered fundamental, or only effective. Under the conditions just outlined, the fit performed reveals a very good quality ($\chi^2/dof = 10.9/10$), corresponding to a 44% probability, and the best values and errors for the main parameters (ℓ_A is fixed to 1.5) are

$$\left\{ \begin{array}{lll} G & = 0.704 \pm 0.002 & [\mathrm{GeV}]^{-1} \\ x & = 0.917 \pm 0.017 & \\ \theta_V & = \quad 31.92 \pm 0.17 & [\mathrm{deg.}] \\ \theta_P & = -11.59 \pm 0.76 & [\mathrm{deg.}] \end{array} \right. \qquad (22)$$

The nonet symmetry breaking parameter is $x = 0.92 \pm 0.02$, confirming a previous analysis [6]. This cannot be changed by leaving free ℓ_A. The value for G is also in nice agreement with the previous analysis of Ref. [6]. The vector mixing angle is found at 3.4 degrees below its ideal value[6] and agrees with predictions [49]. More appealing is the mixing angle of pseudoscalar mesons coming out from fit: $\theta_P = -11.59° \pm 0.76°$, in agreement with the linear mass formula, which predicts $-10.1°$.

4.4. The One Angle η/η' Mixing Scheme from VMD

As discussed above the model we propose, which relies on the VMD approach of Refs. [10,11] with fixed SU(3) breaking à la BKY [12,13], leads to (one angle) formulae for the $\eta/\eta' \to \gamma\gamma$ decay amplitudes. These can be identified with the corresponding Current Algebra standard expressions and we have recalled that they can also be directly

[6]Let us, however, remind that this value relative to ideal mixing is the consequence of our choice $\phi^I = -|s\bar{s}>$.

derived from the WZW Lagrangian. This justifies the identification shown in Eq. (18) for the singlet and octet coupling constants. One should note that nonet symmetry breaking does not modify the formulae substantially. In this case, we obtain together with $\theta_P = -11.59° \pm 0.76°$

$$\frac{f_8}{f_\pi} = 0.82 \pm 0.02 \quad , \quad \frac{f_1}{f_\pi} = 1.15 \pm 0.02 \qquad (23)$$

using Eq. (20), and the fit result for x. Actually, it happens that a low value for f_8/f_π and a low absolute value for the pseudoscalar mixing angle θ_P are correlated properties. Indeed, as can be read off Fig.1 in Ref.[14], a low angle value (in absolute magnitude) implies a low value for f_8/f_π as a consequence of the partial width measured values of $\eta/\eta' \rightarrow \gamma\gamma$. What is interesting, in the VMD approach developed above, is that this result is tightly connected with the BKY breaking, independently of any reference to two–photon decays. Moreover, the value for what *should* be $[f_K/f_\pi]^2$, fit *in radiative decays solely*, confirms this expectation.

Connected with this remark, one can perform a fit of the $VP\gamma$ processes in isolation in order to get estimates for x and θ_P, free of any influence of the $P\gamma\gamma$ processes. This allows to check the concept ual relation between $VP\gamma$ and $P\gamma\gamma$ which can be inferred from VMD. Using the formulae given in the Appendix, one can indeed reconstruct the VMD expectations for the $P\gamma\gamma$ modes. The interesting point here, compared with what is shown in Table 1, is that the fit procedure improve the parameter values associated with the $VP\gamma$ modes. The results are shown in Table 2. An unexpected result here is that the value favored for η radiative decay width is the PDG mean value; this could indicate that $\gamma\gamma$ and Primakoff measurements (which are statistically inconsistent) both suffer from systematic effects, equal in magnitude and opposite in signs. Then, the HLS approach we have developed, even restricted to the $VP\gamma$ processes is indeed able to predict quite nicely the $P\gamma\gamma$ partial widths. The effects of fitting can easily be understood by comparing the corresponding information and accuracies in Tables 1 and 2. Thus, the VMD formulae (which are also those obtained [13] by breaking à la BKY the Wess-Zumino Lagrangian) provide a quite consistent picture. An additional improvement comes from expressing the vector mesons masses and their couplings to e^+e^- as functions of the basic HLS parameters (a, g, f_π), in accordance with the internal structure of the HLS model. One should also note that relatively low values of θ_P have been advocated (or found) in analyzing similar data, for instance in Refs. [14,21,47,48]. It can thus be remarked that in the one mixing angle approach, it is only the addition of J/ψ decays which pushes $|\theta_P|$ to larger values.

As conclusions, within the context of light meson decays, we find no failure of the VMD approach sketched above[7] and no need for a second angle [18,19,23] arises naturally from the data examined so far. However, one cannot exclude that nonet symmetry breaking is somehow equivalent to this specific second angle. This does not seem easy to prove from standard algebra.

5. Is There a Glue Component Coupled to η/η'?

As stated in the Introduction, the precise content of the pseudoscalar singlet component in η/η' mesons is somehow controversial. One cannot indeed exclude the interplay of the usual singlet $\eta_0 = (u\overline{u} + d\overline{d} + s\overline{s})/\sqrt{3}$ with other SU(3) singlet states [14,17], which could be glueballs or some $c\overline{c}$ admixture, or both. Let us assume the existence of such an additional singlet state that will be denoted gg, in order to make formally the connection with its possibly being a gluonium.

5.1. The η/η' Mesons in Terms of Octet and Singlet States

It is thus meaningful to allow for the mixing of $\pi_8 = (u\overline{u} + d\overline{d} - 2s\overline{s})/\sqrt{6}$ with both singlet states referred to just above as η_0 and gg. This follows the proposal in Ref. [14]. We are not actually very dependent on an assumption about the precise content of gg, except that it is supposed orthogonal to η_0. An appropriate parametrization for the mixing of (π_8, η_0, gg) into physical pseudoscalar meson states denoted (η, η', η'') is needed. Using here the symbol η'' for the third partner of the doublet (η, η') simply means that we consider premature to try identifying it, and we do not plan to describe its couplings. Any general parametrization of an orthogonal rotation matrix depends *a priori* on 3 angles. One could for instance choose to express it in terms of the usual Euler angles, however, an appropriate parametrization of this transform is represented by the Cabibbo–Kobayashi–Maskawa matrix (with the complex phase factor δ removed,

[7] The problem with the $K^{*\pm}$ radiative decay can be solved *ad minima* as shown in Ref. [24], and briefly sketched in Section 7.

$\alpha_s = \sin\alpha \; \alpha_c = \cos\alpha, \; \alpha = \beta, \gamma, \theta$)

$$\begin{bmatrix} \eta \\ \eta' \\ \eta'' \end{bmatrix} = \begin{bmatrix} \theta_c\beta_c & -\theta_s\beta_c & \beta_s \\ \theta_s\gamma_c - \theta_c\beta_s\gamma_s & \theta_c\gamma_c + \theta_s\beta_s\gamma_s & \beta_c\gamma_s \\ -\theta_s\gamma_s - \theta_c\beta_s\gamma_c & -\theta_c\gamma_s + \theta_s\beta_s\gamma_c & \beta_c\gamma_c \end{bmatrix} \times \begin{bmatrix} \pi_8 \\ \eta_0 \\ gg \end{bmatrix} \quad (24)$$

Indeed, the vanishing of β and γ gives smoothly the usual mixing pattern of the (η, η') doublet (with $\theta \equiv \theta_P$) and the decoupling of the additional singlet. Setting $\beta = 0$ cancels out glue inside η only, while $\gamma = 0$ removes any glue inside the η' only. Therefore, the transform (24) above allows for analyzing the interplay of an additional singlet (named here glue) in a continuous way for both the η and the η' mesons. The practical use of this formula is obvious : by inverting it, we get expressions for the π_8 and η_0 fields of the matrix P' in terms of η and η' fields ; of course, we can forget about couplings to η''.

5.2. Nonet Symmetry Breaking versus Glue

Up to now, we have illustrated that the BKY breaking was a fundamental tool in order to describe all data concerning radiative and two-photon decays of light mesons. The other central result of our fitting model concerns the unavoidable need of about 10% breaking of nonet symmetry in the pseudoscalar sector ($x \simeq 0.9$). This could well be a fundamental property. However, the observed nonet symmetry breaking could also be an artefact of the model above, reflecting physical effects intrinsically ignored. In this Section we examine the interplay of nonet symmetry breaking and a possible glue component. The $VP\gamma$ coupling constants in this case can be found in Ref. [24] ; we give in the Appendix the formulae for $P\gamma\gamma$ couplings for illustrative purposes (see Rels.(A.5). A phenomenological study of these relations, which include SU(3) breaking, nonet symmetry breaking and glue has been performed with the following conclusions : **1** The BKY breaking is still found determined by the value of f_K/f_π; it can thus be fixed as previously done. **2** Nonet symmetry breaking and glue are intimately connected and reveal a correlation close to the 100% level. This second remark does not mean that nonet symmetry breaking and glue (or any additional singlet) are physically equivalent. The single appropriate conclusion is rather that, in order to conclude firmly about about each of these twin phenomena, one needs relatively precise information on the other.

However, a few additional remarks can be drawn [24]. One can analyze how coupling to glue evolves as a function of a fixed nonet symmetry breaking level. This is shown in Fig 2. One clearly sees there that no need for glue is exhibited by the data if $x \simeq 0.9$ (β and γ can be chosen equal zero without hurting the data). At $x \simeq 1$ and somewhat above, the angle β is still consistent with zero, pointing to the fact that one can hardly claim the need for glue in the η meson. However, somewhat above $x \simeq 0.9$, the level of glue in η' is a rising function of x, as shown by the steep dependence of γ upon x. In order to fix one's idea, if for some reason $x \simeq 1$ has to be prefered, then it implies a sensible content of gl ue in the η'. Writing $\eta' = X\pi_8 + Y\eta_0 + Z(gg)$ (with $X^2+Y^2+Z^2 = 1$), we have $\sqrt{X^2+Y^2} = 0.89$ and $Z = 0.46$. One can then express the glue fraction in η' by $Z^2 = \cos^2\gamma \simeq 0.20$ (at $x = 1$). The correlation between glue and nonet symmetry breaking is not however complete. Below some value ($x \simeq 0.85$), the model fails completely to reach an acceptable probability and no glue contribution (as defined above) helps for recovering. More information can be found in Ref. [24].

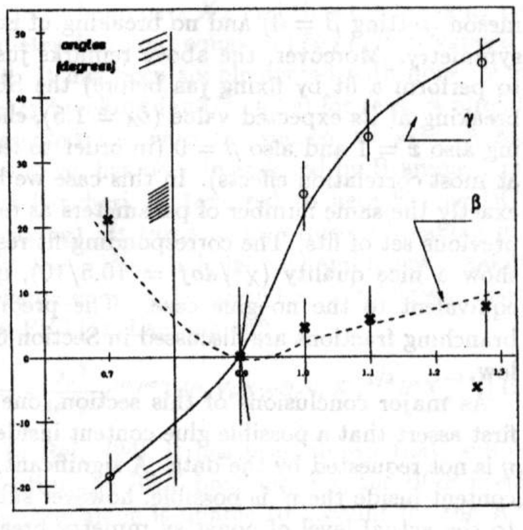

Figure 2. The angles providing the coupling of η and η' to glue (actually, to any singlet state not constituted of u, d and s quarks) as a function of the symmetry breaking parameter x. $x = 1$ corresponds to exact nonet symmetry for the pseudoscalar mesons couplings. A non–zero β is tightly connected with glue in the η meson, while a non–zero γ is tightly connected with glue in the η' meson.

In view of all this, beside the model with no glue and with a small breaking of nonet symmetry, we have studied the case of glue in only the η'

meson (setting $\beta = 0$) and no breaking of nonet symmetry. Moreover, the above remarks justify to perform a fit by fixing (as before) the SU(3) breaking at its expected value ($\ell_A = 1.5$), choosing also $x = 1$ and also $\beta = 0$ (in order to lessen at most correlation effects). In this case we have exactly the same number of parameters as in the previous set of fits. The corresponding fit results show a nice quality ($\chi^2/dof = 10.5/10$), quite equivalent to the no–glue case. The predicted branching fractions are discussed in Section 6 below.

As major conclusions of this section, one can first assert that a possible glue content inside the η is not requested by the data. A significant glue content inside the η' is possible, however subject to the actual level of nonet sy mmetry breaking for which nothing is presently ascertained. We do not discuss any more values and meaning of f_1 and f_8. Eq. (A . 5), indeed shows that the meaning of these has to be revisited. Moreover, the specific two–angle formulation of the $\eta/\eta' \to \gamma\gamma$ decays introduced by the glue coupling (θ_P and γ), appears quite different from the one introduced in Refs. [18,19,23].

6. Estimates for Branching Fractions from Fits

We give and discuss here the reconstruction properties of the two variants of our model, both discussed above. These are **i/** nonet symmetry breaking supplemented by a fixed SU(3) breaking (BKY) and **ii/** a fixed SU(3) breaking (BKY) with glue inside the η' replacing nonet symmetry breaking. We now compare the branching fractions predicted by these two solutions to the accepted branching fractions as given in the PDG [31]. They are computed according to the formulae for coupling constants given in the Appendix and the relations defining the partial widths. The coupling constants just referred to are computed from the basic parameters (G, x, θ_P, θ_V, γ), by identifying these with gaussian distributions having as mean values the central values in the fit and as standard deviations, the corresponding (1σ) error.

In Table T3 we list the information for radiative decays. The first remark which comes to mind by comparing the two model reconstructions is that their predictions are close together (see the first two data column). This illustrates clearly the numerical equivalence of coupling to glue and nonet symmetry breaking. The relative disagreement of $\eta' \to \rho^0\gamma$ with accepted values [31] is actually an interesting artefact. Indeed, what has been submitted to fit is not the branching fraction given in Ref. [31], but the corresponding coupling constant ext racted by the Crystal Barrel Collaboration in [50]. The reason for this is that the (published) branching fraction for $\eta' \to \rho^0\gamma$ is influenced by a non–resonant contribution originating from the box anomaly [6,43,16,44,45] for the vertex $\eta'\pi^+\pi^-\gamma$. This is not accounted for in the VMD model of [11] and has thus to be removed. Actually this process contributes to the total χ^2 by only $\simeq 0.5$. Moreover, its importance is not that decisive that it influences the fit results dramatically. This last information, which has been tested by removing the $\eta' \to \rho^0\gamma$ decay from fit data, substantiates the relevance of describing [6] this decay mode by a coherent sum of a resonant (ρ) contribution and a non–resonant contribution, whatever is the precise meaning of this last (phase space) contribution [6]. This result also shows that the corresponding coupling constant is indeed consistent with the estimates of Refs. [6,50], which exhibited a weak model dependence with respect to various ρ^0 resonance lineshapes.

On the other hand, even if quite acceptable, the reconstruction for $\eta \to \gamma\gamma$ branching fraction is influenced by having used for this decay mode the mean value of the measurements obtained in $\gamma\gamma$ experiments, while the PDG information reported for $\eta \to \gamma\gamma$ branching fraction is the official one [31], somehow influenced by the Primakoff measurement. We have already commented on the result for this decay width in relation with Table 2. The single clear disagreement of model predictions with data concerns the branching fraction for $K^{*\pm} \to K^{\pm}\gamma$, that we find about half of the reported value in PDG [31]. We postpone to Section 7 the reexamination of this question.

Otherwise, the largest disagreement is never greater than about 1.5σ. At such a (non–significant) level, it is hard to distinguish whether differences between predictions and data are due to SU(2) breaking effects missing in the models, to systematic errors in the data or to the (unavoidable) influence of the resonance models used to extract branching fractions from data. For instance, changing the model for the ρ lineshape [30] in the cross section for $e^+e^- \to \pi^0\gamma$ allows to reduce the branching ratio for $\rho^0 \to \pi^0\gamma$ from (6.8 ± 1.7) 10^{-4} to (6.1 ± 1.5) 10^{-4} which compares better to the corresponding prediction ($5.2\ 10^{-4}$).

The recent measurements for $\phi \to \eta'\gamma$ are also well accepted by the fit. However, the prediction tends to indicate that the central value found by SND [38] is favored compared to that of the CMD-2 [37]. All this leads us to conclude that the model of symmetry breaking we have presented provides a consistent description of the data. At their present level of accuracy, these do not seem to require additional symmetry breaking. An especially satisfactory conclusion is that SU(3) breaking effects are not left free in the fits and are practically determined by the ratio f_K/f_π. Some nonet symmetry breaking in the pseudoscalar sector is, however, requested by the data. This is fully or partly degenerated with a possible admixture of glue, shown to (possibly) affect only the η' meson. If this has to be seriously considered, the question is to identify the third partner to the (η, η') doublet which has been named η''. For this purpose, a precise study of the decay properties of the $\eta(1440)$ meson could improve the hint. One has also to remind that this glue component could actually be a $c\bar{c}$ admixture, even if 20% admixture could look a little bit too much.

7. The $K^{*\pm}$ Radiative Decay Problem

As shown by the two leftmost data columns in Table 3, the two variants of the model presented above do not account for the accepted [31] radiative decay width $K^{*\pm} \rightarrow K^\pm \gamma$. One cannot exclude that this measurement might have to be improved by other means that the Primakoff effect. However, this decay mode has been measured separately for the two charged modes and found to agree with each other. Therefore, the possibility that this failure indicates that models have to be refined cannot be avoided. The first point which comes to mind is whether the disagreement reported above (a factor of two between prediction and measurement) could be attributed to (missing) SU(2) flavor symmetry breaking effects. If one takes into account the quark content of the K^*'s, the answer is seemingly no. Indeed, in this case, one could guess that significant unaccounted for SU(2) breaking effects would rather affect the quality of predictions for K^{*0} rather than for $K^{*\pm}$. However, the absolute partial width of the K^{*0} is well predicted by our modellings (flavor SU(3) and nonet symmetry breakings or glue). This possibility seeming unlikely, the question becomes : can the VMD modelling we developed be modified in order to account for this mode within an extended SU(3) breaking framework? The reply is positive and is the following.

7.1. The K^* Model

Within the spirit of the BKY mechanism, the (unbroken) FKTUY Lagrangian given in Rel. (4) can be broken straightforwardly in three different ways. The first mean is the pseudoscalar field renormalization (see Rel. (13)), which leads to introduce the matrix X_A and thus the breaking parameter ℓ_A found equal to $(f_K/f_\pi)^2$ as expected [12]. It has been supplemented with nonet symmetry breaking (and/or glue) for reasons already presented and with the success we saw. A second mean has been introduced by Bramon, Grau and Pancheri [7] (referred to hereafter as BGP breaking). It turns to introduce a breaking matrix $X_W = \mathrm{diag}(1, 1, 1+c_W)$ and a new breaking parameter $\ell_W = 1+c_W$ in a symmetric way inside the FKTUY Lagrangian[8] :

$$L = -\frac{3g^2}{4\pi^2 f_\pi}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[\partial_\mu V_\nu X_W \partial_\rho V_\sigma X_A^{-1/2} P' X_A^{-1/2}] \quad (25)$$

In Ref. [24], it has been shown that, supplementing the BKY breaking X_A, the BGP breaking X_W alone is unable to account for the $K^{*\pm}$ radiative decays. Moreover, the constant ℓ_W is pushed to 1 by the fit procedure, and then to no BGP breaking ($c_W = 0$). A third mean is however conceivable. One should note that the BKY breaking mechanism [12] implies a renormalization (or redefinition) of the pseudoscalar field matrix expressed through X_A ; however, the X_V breaking does not end up with a renormalization of the vector field matrix, which remains unchanged in the breaking procedure. One can then *postulate* that the vector meson field matrix has also to be SU(3) broken and also in a symmetric way. This is done by performing the change :

$$V \longrightarrow X_T V X_T \ , \quad [X_T = \mathrm{diag}(1, 1, 1+c_T)] \quad (26)$$

in Rel. (25), in complete analogy with the renormalization of the P matrix. A lack of fancy (not still a mathematical proof) seems to indicate that no fourth mechanism can play. A detailed study of the consequences of Lagrangian (26) has been performed in Ref. [24] with an interesting conclusion : if one fixes $\ell_A = 1.5$ as expected [12,13], the X_T and X_W breaking are so sharply correlated that they cannot be left free together. More interestingly, it was found phenomenologically that the additional breaking parameters fulfill :

$$(1 + c_W)(1 + c_T)^4 = 1 \quad (27)$$

This tells us that the most general form of the broken FKTUY Lagrangian accepted by the data can be symbolically written :

$$L = C\, \mathrm{Tr}[(X_T^{-1} V X_T)(X_A^{-1/2} P' X_A^{-1/2})(X_T V X_T^{-1})] \quad (28)$$

In this case, all couplings constants write as when having solely the BKY breaking mechanism, supplemented with nonet symmetry breaking and/or

[8] The symmetry can be made manifest. What is written in Rel. (25), can be symbolically written $\mathrm{Tr}[V X_W V (X_A^{-1/2} P X_A^{-1/2})]$ and is obviously identical to $\mathrm{Tr}[X_W^{1/2} V (X_A^{-1/2} P X_A^{-1/2}) V X_W^{1/2}]$.

glue, except for the K^* decay modes which become :

$$\begin{cases} G_{K^{*0}K^0\gamma} = & - \quad G\dfrac{\sqrt{K'}}{3}(1+\dfrac{1}{\ell_T}) \\ G_{K^{*\pm}K^\pm\gamma} = & \quad G\dfrac{\sqrt{K'}}{3}(2-\dfrac{1}{\ell_T}) \end{cases} \tag{29}$$

where $K' = \ell_T/\ell_A$ and $\ell_T = (1+c_T)^2$. Stated otherwise, both K^* couplings are changed : the one correctly accounted for by the previous modellings (K^{*0}), *and* the one poorly described ($K^{*\pm}$). Therefore, a fit value for ℓ_T must change $G_{K^{*\pm}}$ while leaving $G_{K^{*0}}$ practically unchanged, despite the *functional* relation among them. Assuming no coupling to glue, we have performed the fit and found a perfect fit quality ($\chi^2/dof = 11.07/10$) with practically the same parameter values as in the models above and additionally :

$$\ell_T = 1.19 \pm 0.06 \quad , \quad (c_T = 0.109 \pm 0.024) \tag{30}$$

The predicted branching fractions are given in the third data column in Table **3**. They indeed show that all predictions (including for the K^{*0} mode) are unaffected except for the $K^{*\pm}$ mode, now in quite nice agreement with its accepted value [31]. One may wonder that the K^{*0} mode is unchanged, while the $K^{*\pm}$ mode is increased by a factor $\simeq 2$. For this purpose, one may compare the values of the ℓ_T part of the couplings in Rels. (29) at $\ell_T = 1$ and at $\ell_T = 1.2$. One thus find that the former change is $2 \rightarrow 2.01$, while the later change is $1 \rightarrow 1.28$. Therefore, the change requested in order to account for the $K^{*\pm}$ mode, results in an unsignificant change for the K^{*0} mode. Therefore, quite unexpectedly, a tiny change in the VMD model we have shown is enough to describe indeed all radiative decays at their presently accepted values. Nevertheless, the additional mechanism complicates the full breaking picture which is otherwise quite simple. One can hope that new measurements for the $K^{*\pm}$ radiative decay will come soon and tell definetely whether the picture has really to undergo this complication. We do not discuss here, the correlation between glue component and nonet symmetry, all conclusions reported above remain fully valid.

7.2. The K^* Model and the WZW Lagrangian

In Section 3.4, we have remarked that imposing the change of fields from P to P' to the WZW Lagrangian (see Rel.(19)) provides the same description of radiative decays of pseudoscalar mesons than the broken HLS–FKTUY model. Thus, one has checked that that these two descriptions were indeed equivalent. The VMD description is however able to connect the $P\gamma\gamma$ couplings to the $VP\gamma$ ones with the success illustrated by Table **2**, while nothing analogous can be inferred from the WZW Lagrangian. When introducing the additional breaking schemes in order to construct the K^* model sketched above (the expanded Lagrangian can be found in the Appendix of Ref. [24]), this property is formally lost, except if additionally to the change $P \rightarrow P'$, we also perform the change $Q^2 \rightarrow X_W X_T^4 Q^2$, e.g. if we "renormalize" the SU(3) charge matrix, or the WZW Lagrangian as a whole. It happpens that the condition in Rel. (27) prevents such an ugly transform. Stated otherwise, phenomenology forces a relation which is such that the two–photon decays of pseudoscalar mesons are still given by the Wess–Zumino–Witten Lagrangian [44,?], with breaking only for the single occuring matter field matrix. In order that the equivalence between VMD and WZW is generally maintained, the K^* breaking should affect the K^* couplings only. If instead the other couplings $VP\gamma$ were affected, this would propagate down to the $P\gamma\gamma$ couplings. In this case, the equivalence statement between (broken) VMD and (broken) WZW would no longer hold. Thus, the K^* breaking scheme seems indeed to be the most general consistent with this equivalence statement.

8. Conclusion

We have presented a VMD based model aiming at describing the radiative decays of light flavor mesons, including the two–photon decays of pseudoscalar mesons. This model relies heavily on the HLS model supplemented with the BKY breaking mechanism in order to account for SU(3) symmetry breaking. In order that this model gives a satisfactory account of all observables it can structurally cover, something additional has to be introduced.

As a first way out, we have shown that a breaking of nonet symmetry for pseudoscalar mesons (only) was able to explain all data, including two–photon decays.

A second possibility has been to modify the singlet sector of the pseudoscalar mesons, by allowing the coupling of the η/η' sector to a component named glueball, which could also be a $c\bar{c}$ admixture. Actually, radiative decays of light mesons alone cannot provide a detailed information about the possible content of this possible additional singlet.

We have also shown that, if nonet symmetry breaking and additional glue (or $c\bar{c}$) admixture can coexist, it is impossible to share the influence of each. This can only come from external (experimental or theoretic) information. One can however assert that the coupling of this additional singlet state to the η meson is not requested by the data. Instead, depending on the actual level of nonet symmetry breaking, this additional singlet can represent up to about 20% in the η' meson.

The breaking mechanism proposed by Bando, Kugo and Yamawaki is mandatory in the problem of radiative decays. It should be stressed that phenomenology indeed confirms the connection of the corresponding breaking parameter with the ratio f_K/f_π.

The picture that emerges from there is quite consistent and tends to indicate that present data do not require any breaking of the SU(2) symmetry at a visible level in only radiative decays of light mesons.

The single present data which requires a special additional input is the $K^{*\pm}$ radiative decay. It can be done succesfully without destroying an equivalence statement between VMD and the WZW description of pseudoscalar meson decays to two photons. However, a confirmation of the present data for the $K^{*\pm}$ radiative decay looks desirable.

Anyway, whatever is the precise value of the $K^{*\pm}$ radiative width, VMD expressed through the general concept underlying the HLS model is able to provide a quite consistent picture of the radiative decays of light mesons.

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1. Appendix A

1.1. Matrix Elements with SU(3) and Nonet Broken Symmetries

In terms of the angles (δ_V, δ_A) of physical states with respect to ideal mixing, as defined in Section 3, the coupling constants at vertices $VP\gamma$ which can be deduced from the Lagrangian in Rel.(13) are:

$$\left\{\begin{array}{lcl}
G_{\rho^0\pi^0\gamma} & = & \frac{1}{3}G \\
G_{\rho^\pm\pi^\pm\gamma} & = & \frac{1}{3}G \\
G_{K^{*0}K^0\gamma} & = & -G\frac{2\sqrt{Z}}{3} \\
G_{K^{*\pm}K^\pm\gamma} & = & G\frac{\sqrt{Z}}{3} \\
G_{\rho^0\eta\gamma} & = & \frac{1}{3}G\left[\sqrt{2}(1-x)\cos\delta_P - (2x+1)\sin\delta_P\right] \\
G_{\rho^0\eta'\gamma} & = & \frac{1}{3}G\left[\sqrt{2}(1-x)\sin\delta_P + (2x+1)\cos\delta_P\right] \\
G_{\omega\pi^0\gamma} & = & G\cos\delta_V \\
G_{\phi\pi^0\gamma} & = & -G\sin\delta_V
\end{array}\right. \qquad (A.1)$$

$$\left\{\begin{array}{lcl}
G_{\omega\eta\gamma} & = & \frac{1}{9}G\left[-(2x+1)\cos\delta_V\sin\delta_P - 2Z(2+x)\sin\delta_V\cos\delta_P + 2\sqrt{2}Z(1-x)\sin\delta_V\sin\delta_P + \sqrt{2}(1-x)\cos\delta_V\cos\delta_P\right] \\
G_{\omega\eta'\gamma} & = & \frac{1}{9}G\left[(2x+1)\cos\delta_V\cos\delta_P - 2Z(2+x)\sin\delta_V\sin\delta_P - 2\sqrt{2}Z(1-x)\sin\delta_V\cos\delta_P + \sqrt{2}(1-x)\cos\delta_V\sin\delta_P\right] \\
G_{\phi\eta\gamma} & = & \frac{1}{9}G\left[(2x+1)\sin\delta_V\sin\delta_P - 2Z(2+x)\cos\delta_V\cos\delta_P + 2\sqrt{2}Z(1-x)\cos\delta_V\sin\delta_P - \sqrt{2}(1-x)\sin\delta_V\cos\delta_P\right] \\
G_{\phi\eta'\gamma} & = & -\frac{1}{9}G\left[(2x+1)\sin\delta_V\cos\delta_P + 2Z(2+x)\cos\delta_V\sin\delta_P + 2\sqrt{2}Z(1-x)\cos\delta_V\cos\delta_P + \sqrt{2}(1-x)\sin\delta_V\sin\delta_P\right]
\end{array}\right. \qquad (A.2)$$

where the SU(3) breaking parameter comes through $Z = 1/\ell_A$. The dependence upon the nonet symmetry breaking parameter x is explicit. The basic parameter G yields the following expression:

$$G = \frac{Ce}{2g} = -\frac{3eg}{8\pi^2 f_\pi} \qquad (A.3)$$

which depends on the vector meson universal coupling g and the pion decay constant f_π. We will fit the absolute value of G. Correspondingly the matrix elements for the decays $\pi^0/\eta/\eta' \to \gamma\gamma$ are :

$$\begin{cases} G_{\eta\gamma\gamma} = & -\frac{\alpha_{em}}{\pi\sqrt{3}f_\pi}\left[\frac{5-2Z}{3}\cos\theta_P - \right. \\ & \left. -\sqrt{2}\frac{5+Z}{3}x\sin\theta_P\right] \\ G_{\eta'\gamma\gamma} = & -\frac{\alpha_{em}}{\pi\sqrt{3}f_\pi}\left[\frac{5-2Z}{3}\sin\theta_P + \right. \\ & \left. +\sqrt{2}\frac{5+Z}{3}x\cos\theta_P\right] \\ G_{\pi^0\gamma\gamma} = & -\frac{\alpha_{em}}{\pi f_\pi} \end{cases} \quad (A.4)$$

1.2. $\eta/\eta' \to \gamma\gamma$ Decays With Coupling to Glue

In this section, we display the expression of the coupling constants for $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ transitions. The angles β and γ have been defined by Rel. (24). The matrix element for the decay $\pi^0 \to \gamma\gamma$ is not affected by the a dditional singlet (named glue in the body of the text). The amplitudes for $\eta/\eta' \to \gamma\gamma$ are :

$$\begin{cases} G_{\eta\gamma\gamma} = & -\frac{\alpha_{em}\cos\beta}{\pi\sqrt{3}f_\pi}\left\{\frac{5-2Z}{3}\cos\theta_P - \right. \\ & \left. -\sqrt{2}\frac{5+Z}{3}x\sin\theta_P\right\} \\ & -\frac{\alpha_{em}\cos\gamma}{\pi\sqrt{3}f_\pi}\left\{\left[\frac{5-2Z}{3} + \right.\right. \\ G_{\eta'\gamma\gamma} = & +\sqrt{2}\frac{5+Z}{3}x\sin\beta\tan\gamma]\sin\theta_P + \\ & +[\sqrt{2}\frac{5+Z}{3}x - \\ & \left. -\frac{5-2Z}{3}\sin\beta\tan\gamma]\cos\theta_P\right\} \end{cases} \quad (A.5)$$

Table 1
Partial decay widths of the pseudoscalar mesons, as reconstructed from VMD, using the $VP\gamma$ measured couplings, and their direct accepted measurements [31].

Mode	VMD prediction	PDG
$\pi^0 \to \gamma\gamma$[eV]	10.73 ± 1.20	7.74 ± 0.50
$\eta \to \gamma\gamma$[keV]	0.62 ± 0.18	0.46 ± 0.04
$\eta' \to \gamma\gamma$[keV]	5.10 ± 0.76	4.27 ± 0.19

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Table 2
Partial decay widths of the η/η' mesons, as reconstructed from fit to solely the radiative decays $VP\gamma$ (leftmost data column) and their direct measurements [31] (rightmost data column).

Mode	VMD Fit	PDG	Comment
		0.514 ± 0.026	$\gamma\gamma'$
$\eta \to \gamma\gamma$ [keV]	0.464 ± 0.026	0.46 ± 0.04	PDG mean
		0.324 ± 0.046	Primakoff
$\eta' \to \gamma\gamma$[keV]	4.407 ± 0.233	4.27 ± 0.19	PDG mean

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Table 3

Branching fractions from fits for radiative decays under various conditions of symmetry breakings. Note that the rate for $K^{*\pm}$ is a prediction in the first two data columns, while the corresponding data is included in the fit which leads to the third data column.

Process	Nonet Sym. + SU(3)	Glue + SU(3)	$K^{*\pm}$ Breaking	PDG
$\rho \to \pi^0\gamma$ $(\times 10^4)$	5.16 ± 0.03	5.16 ± 0.03	5.16 ± 0.03	6.8 ± 1.7
$\rho \to \pi^\pm\gamma$ $(\times 10^4)$	5.12 ± 0.03	5.12 ± 0.03	5.12 ± 0.03	4.5 ± 0.5
$\rho \to \eta\gamma$ $(\times 10^4)$	3.25 ± 0.10	3.28 ± 0.10	3.31 ± 0.09	$2.4^{+0.8}_{-0.9}$
$\eta' \to \rho\gamma$ $(\times 10^2)$	33.1 ± 2.0	33.7 ± 2.0	33.0 ± 1.8	30.2 ± 1.3
$K^{*\pm} \to K^\pm\gamma$ $(\times 10^4)$	5.66 ± 0.03	5.66 ± 0.03	9.80 ± 0.93	9.9 ± 0.9
$K^{*0} \to K^0\gamma$ $(\times 10^3)$	2.30 ± 0.01	2.30 ± 0.01	2.32 ± 0.02	2.3 ± 0.2
$\omega \to \pi^0\gamma$ $(\times 10^2)$	8.50 ± 0.05	8.50 ± 0.05	8.50 ± 0.05	8.5 ± 0.5
$\omega \to \eta\gamma$ $(\times 10^4)$	8.0 ± 0.2	8.1 ± 0.2	8.12 ± 0.19	6.5 ± 1.0
$\eta' \to \omega\gamma$ $(\times 10^2)$	2.8 ± 0.2	2.9 ± 0.2	2.8 ± 0.2	3.01 ± 0.30
$\phi \to \pi^0\gamma$ $(\times 10^3)$	1.27 ± 0.13	1.28 ± 0.12	1.26 ± 0.13	1.31 ± 0.13
$\phi \to \eta\gamma$ $(\times 10^2)$	1.25 ± 0.04	1.25 ± 0.05	1.22 ± 0.04	1.26 ± 0.06
$\phi \to \eta'\gamma$ $(\times 10^4)$	0.61 ± 0.027	0.55 ± 0.03	0.63 ± 0.02	$1.2^{+0.7}_{-0.5}$
$\eta \to \gamma\gamma$ $(\times 10^2)$	40.5 ± 1.7	40.8 ± 1.8	41.5 ± 1.4	39.21 ± 0.34
$\eta' \to \gamma\gamma$ $(\times 10^2)$	2.1 ± 0.1	2.1 ± 0.1	2.1 ± 0.1	2.11 ± 0.13

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